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[This question paper contains 4 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : **851** **G**

Unique Paper Code : 2352572301

Name of the Paper : Differential Equations

Name of the Course : **B.Sc. (Physical Science and Mathematical Science) with Operational Research and Bachelor of Arts**

Semester : III

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **all** questions by selecting **two** parts form each question.
3. **All** questions carry equal marks.

1. (a) Find the general solution of the Bernoulli equation given by $\frac{dy}{dx}+\frac{y}{2x}=\frac{x}{y^3}$ with initial condition $y(1)=2$. Also find an integrating factor for the linear differential equation

$$\frac{dy}{dx}+\left(\frac{2x+1}{x}\right)y=e^{-2x}.$$ (7½)

P.T.O.

(b) Find the general solution of the differential equation $(x^2 - 3y^2)dx + 2xy\,dy = 0$ by showing it's a homogeneous equation. Also show that $M(tx, ty) = tM(x, y)$ for $M = y + \sqrt{x^2 + y^2}$. (7½)

(c) Determine the most general $N(x,y)$ for the equation $(x^{-2}y^{-2} + xy^{-3})dx + N(x, y)dy = 0$ such that the equation is exact and solve the resulting exact equation. (7½)

2. (a) Assume that the population of a certain city increase at a rate proportional to the number of inhabitants at any time, if the population doubles in 40 years, in how many years will it triple? (7½)

(b) Show that the relation $x^2 + y^2 - 25 = 0$ is an implicit solution of the differential equation $x + y\frac{dy}{dx} = 0$ on the interval $-5 < x < 5$. Explain whether the relation $x^2 + y^2 + 25$ is also an implicit solution of $x + y\frac{dy}{dx} = 0$. (7½)

(c) Find the particular solution of the linear system that satisfies the stated initial conditions :

$$\frac{dy_1}{dt} = y_1 + y_2, \; y_1(0) = 1$$

$$\frac{dy_2}{dt} = 4y_1 + y_2, \; y_2(0) = 6. \qquad (7½)$$

3. (a) Solve the initial value problem :
$x^2y'' + 3xy' + y = 0, \; y(1) = 4, \; y'(1) = -1.$ (7½)

(b) Find a homogeneous linear ordinary differential equation for which two functions x^{-3} and $x^{-3} \ln x \; (x > 0)$ are solutions. Show also linear independence by considering their Wronskian. (7½)

(c) Consider the initial value problem :

$$\frac{dy}{dx} = 1 + y^2, \; y(0) = 0.$$

Examine the existence and uniqueness of solution in the rectangle : $|x| < 5, \; |y| < 3.$ (7½)

4. (a) Find a general solution of the following nonhomogeneous differential equation :
$y'' + 4y' + 4y = e^{-2x} \sin 2x.$ (7½)

(b) Use the method of undetermined coefficients to find the particular solution of the differential equation :
$y'' - 2y' + y = x^2 + e^x.$ (7½)

(c) Use the method of variation of parameters to find a particular solution of the differential equation : $y'' + y = \tan x \sec x$. (7½)

5. (a) Find the general solution of the equation.
$$(x-y)y^2u_x + (x-y)x^2u_y = (x^2+y^2)u$$ (7½)

(b) Eliminate the constants a and b from the equation
$$2z = (ax+y)^2 + b$$ (7½)

(c) Solve the initial value problem :
$$u_t + uu_t = x, \quad u(x,0) = 1$$ (7½)

6. (a) Find the general solution of the linear partial differential equation.
$$x(y^2-z^2)u_x + y(z^2-x^2)u_y + z(x^2-y^2)u_z = 0$$
(7½)

(b) Use $v = \ln u$ and $v = f(x) + g(y)$ to solve the equation.
$$x^2u_x^2 + y^2u_y^2 = u^2$$ (7½)

(c) Reduce the equation: $x^2u_{xx} + 2xu_{xy} + y^2u_{yy} = 0$ to canonical form and hence find the general solution. (7½)